Part II:

Interactions of ionizing radiations with matter

Chapter I:
Interactions of charged particles with matter: basic considerations

Introduction

Ionizing radiations:

- Charged particles: - e^- and e^+
  - Light ions (muons, protons, deutons, α)
  - Heavy ions
  - Structured objects
- Photons: - γ-rays (nuclear origin)
  - X-rays (atomic origin)
- Neutrons

Interactions in matter (1)

- For each radiation → particular interaction mechanism with matter (collection of isolated atoms without interaction between them → atoms « gas »):

- For charged particles: Coulomb interactions with electrons and nuclei → frequent collisions → Energy loss in a quasi-continuous way → finite distance in matter → definition of a range. Charged particles are direct ionizing radiations.

- For photons and neutrons: no charge → other mechanisms → no interaction between 2 « catastrophic » events → probability to go through material without interaction → no definition of range. They are indirectly ionizing radiations

Interactions in matter (2)

The interactions of radiations with matter can change the state of the radiation or of the matter

→ for radiation: can be absorbed, can be deviated, can lose energy, can be modified (ex: α → He^{+})

→ for matter: atoms or molecules can be excited, ionized or nuclear reactions are possible. Excitations or ionizations are followed by various processes (chemical reactions,…)

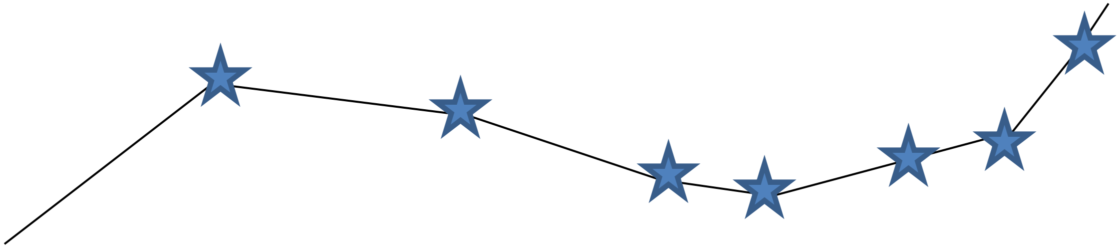
⇒ We will see most probable interactions of radiations in matter

Interaction of charged particles with matter

Charged particle undergoes Coulomb collisions (nuclear reactions are not considered) with:

- nucleus (rare): large energy loss and large angular deviation (elastic collisions: only kinetic energy is transferred)

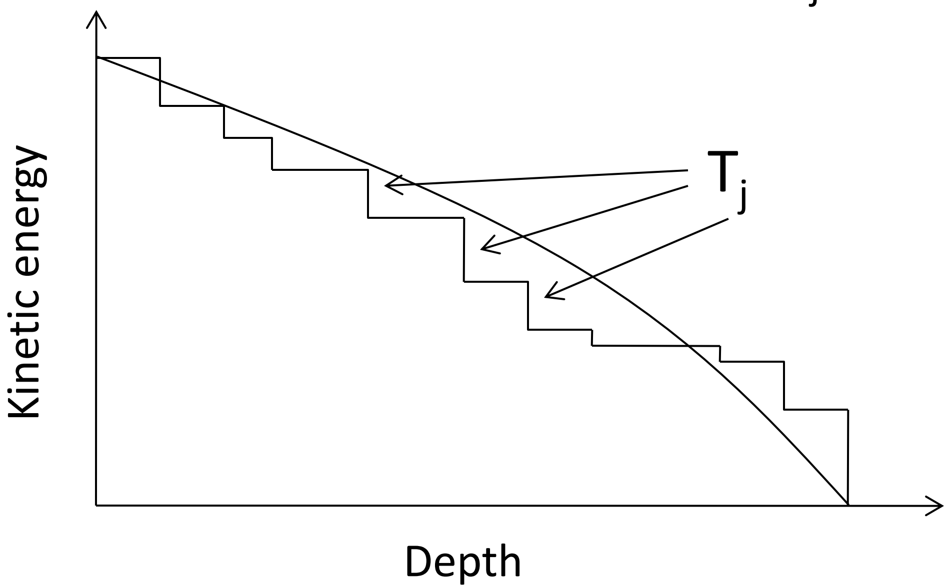
- electrons (frequent) : excitations or ionizations → small energy loss and small angular deviation (inelastic collisions: potential energy is transferred)



Trajectory of the particle

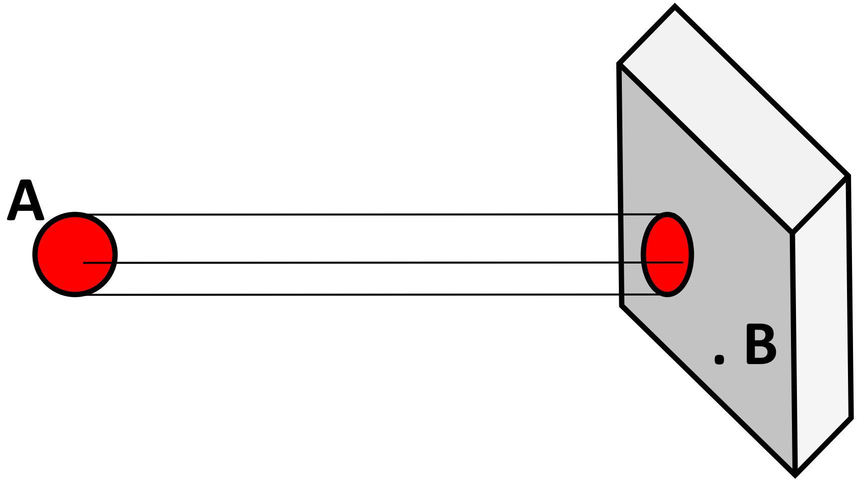
Variation of energy along trajectory

For each collision → energy loss T_j



Multiple collisions → random process → two projectiles imply two different stories → huge number of collisions → small fluctuations → definition of general quantities is possible

Definition of cross section



Consider a particle A of radius r and thus of section $s=\pi r^2$ and a particle B assimilated to a point somewhere in a target of 1 m^2. The probability to interact is the probability that B is inside an area *s* on the target of 1 m^2 → It is the ration between both area.
s is called cross section (unit: m^2 or barn – 1 barn=10^{-28} m^2) →

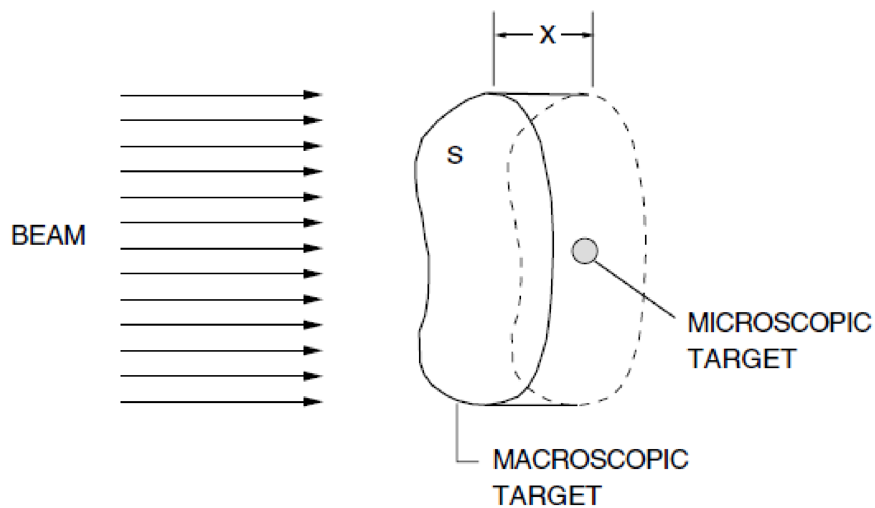
Cross section: Fictitious area that must have an incident particle to reproduce the observed probability of collision or reaction with an incident particle

Types of cross section

- Definition of a collision → interaction between an incident particle and a target particle that implies a measurable specific effect → the value of the cross section does not only depend on incident and target particles and on their relative velocity but also on the type of considered physical effect → cross sections of diffusion, absorption, ionization, excitation,…
- Large number of types of incident particles target particles → large number of types of interactions
- It is possible to look for a particular energy loss (if E is lost) or for a particular direction of emission (if the direction is modified) → differential cross section in energy or in angle ($d\sigma/dE$, $d\sigma/d\Omega$)

Statistical definition of the cross section

- Impossible to experimentally determine microscopic cross sections by bombarding an atom with only one particle → use of statistical information obtained from a large number of bombardments (beam) on a macroscopic target (medium)

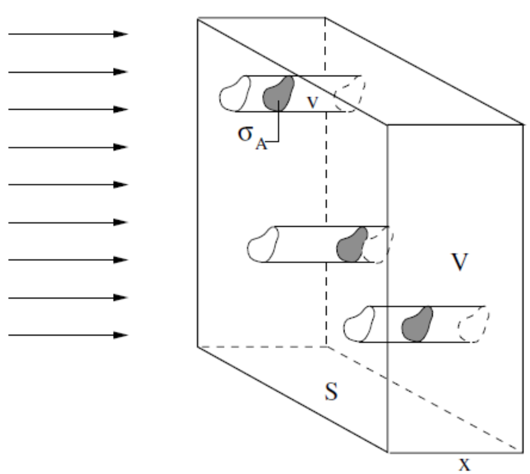


- Projectiles of the beam only interact with medium and not between them

Cross section ↔ probability

If we consider a beam with a density of current *J* (unit: number of part. × s^{-1} × m^{-2}), a target of area *S* (smaller than the beam area) and an interaction process *A* with cross section σ_A → we obtain n_A, the number of interactions *A* induced by the beam particles per time unit:

$$n_A = JS \times \frac{\sigma_A}{S} = J\sigma_A$$



For a volume *V* = *Sx* and a density of target particles *N* (N≈10^{23} atoms/cm^3), n_A →

$$n_A = N \times Sx \times J\sigma_A = JS \times Nx\sigma_A$$
and
$$P_A = Nx\sigma_A \quad \text{for} \quad Nx\sigma_A \ll 1$$

P_A: Probability for the projectile to undergo process *A*

Multiple collisions (1)

- If $Nx\sigma_A$ is not small → multiple collisions are possible
- P_n: probability for the projectile to initiate precisely n events A → equivalent to have n target particles in the cylinder of volume $v = x\sigma_A$ associated to one trajectory → standard problem of gas kinetic theory
- P_n follows Poisson distribution →

$$P_n = \frac{(Nv)^n}{n!} e^{-Nv}$$

$$\Rightarrow \langle n \rangle = Nv = Nx\sigma_A$$

Multiple collisions (2)

$$\Rightarrow P_0 = e^{-Nx\sigma_A}$$

Lambert & Beer's law that governs absorption phenomena

$$\text{If } Nx\sigma_A \ll 1$$

$$P_n \simeq \begin{cases} 1 - Nx\sigma_A & \text{for} \quad n = 0 \\ Nx\sigma_A & \text{for} \quad n = 1 \\ 0 & \text{for} \quad n \geq 2 \end{cases}$$

Mean free path

Mean free path λ_A: Mean distance between 2 processes of type A

$$\lambda_A = \frac{1}{N\sigma_A}$$

For multiple processes A,B,C,…→

$$\sigma_{total} = \sigma_A + \sigma_B + \sigma_C + \ldots$$

$$\frac{1}{\lambda_{total}} = \frac{1}{\lambda_A} + \frac{1}{\lambda_B} + \frac{1}{\lambda_C} + \ldots$$

Stopping power

The (linear) stopping power (more correct: stopping force) of a charged particle with kinetic energy *E* in a medium is mean energy loss (ΔE) per unit length undergone by the particle along its trajectory (Δx) (unit: Jm^{-1} or eVm^{-1}) → $\Delta E/\Delta x$

Mathematical expression of the stopping force (1)

- Consider a target with depth *Δx* (small by comparison to the penetration length of the particle)
- Consider a projectile with energy *E*
- The losses of energy are discrete values T_j with *j=1,2,...* and $T_j \ll E \; \forall \; j$

$$\Rightarrow \Delta E = \sum_j n_j T_j$$ with n_j the number of collisions of type j

$$\Rightarrow \langle \Delta E \rangle = \sum_j \langle n_j \rangle T_j$$ with $\langle n_j \rangle = N \Delta x \sigma_j$

$$\Rightarrow \langle \Delta E \rangle = N \Delta x \sum_j T_j \sigma_j$$

Mathematical expression of the stopping force (2)

$$S = \sum_j T_j \sigma_j$$: Stopping cross section

$$\frac{\langle \Delta E \rangle}{\Delta x} = NS = N \sum_j T_j \sigma_j$$: Stopping force or Stopping power

- Stopping force → macroscopic quantity
- Stopping cross section → microscopic quantity
- Attention: confusion between these 2 quantities (different units)

Straggling parameter (1)

- Mean quadratic fluctuations in energy are →

$$\Omega^2 = \overline{(\Delta E - \langle \Delta E \rangle)^2}$$

- By considering:

$$\Delta E - \langle \Delta E \rangle = \sum_j (n_j - \langle n_j \rangle) T_j$$

- We obtain →

$$\overline{(\Delta E - \langle \Delta E \rangle)^2} = \sum_{j,l} \overline{(n_j - \langle n_j \rangle)(n_l - \langle n_l \rangle)} T_j T_l$$

Straggling parameter (2)

- For *j* = *l* → properties of the Poisson distribution →

$$\overline{(n_j - \langle n_j \rangle)^2} = \langle n_j \rangle = N\Delta x \sigma_j$$

- For *j* ≠ *l* → we transform the expectation value of the product into the product of the expectation values (statistical independence between the ≠ types of collision) →

$$\overline{(n_j - \langle n_j \rangle)(n_l - \langle n_l \rangle)} = \overline{(n_j - \langle n_j \rangle)} \times \overline{(n_l - \langle n_l \rangle)}$$

as $\overline{n_j - \langle n_j \rangle} = 0$ → all terms with *j* ≠ *l* are 0

Straggling parameter (3)

- We thus obtain

$$\Omega^2 = \sum_j \langle n_j \rangle T_j^2 = N\Delta x \sum_j T_j^2 \sigma_j = N\Delta x W$$

with *W*, the straggling parameter that characterizes the fluctuations in energy and is defined as (microscopic parameter):

$$W = \sum_j T_j \sigma_j^2$$

Integral notation

- Continuous spectrum in energy losses (ionizing collisions,…) →

$$\sigma_j \to \frac{d\sigma}{dT} \Delta T_j$$

- With ΔT_j small enough → additions are replaced by integrations →

$$\begin{aligned} S &= \int T d\sigma \\ W &= \int T^2 d\sigma \end{aligned}$$

with $d\sigma = \frac{d\sigma}{dT} dT$

Thick target

- Up to now *Δx* small → *E* constant
- In a general way *S* et *W* are depending on *E*
- If we consider that fluctuations of energy losses can be neglected → the energy *E* of the projectile is a well-defined function of the penetration thickness *x* → *E = E(x)* → « Continuous Slowing Down Approximation » - CSDA →

$$\frac{dE}{dx} = -NS(E)$$

- Sign - takes into account the decrease of the projectile energy

Range of a charged particle

The range *R* of a charged particle of energy *E* in a medium is the average path length $\langle l \rangle$ traveled by the particle as it slows down to rest (without considering thermic motion)

In CSDA ⇒ $$x = \int_{E(x)}^{E_0} \frac{dE'}{NS(E')}$$

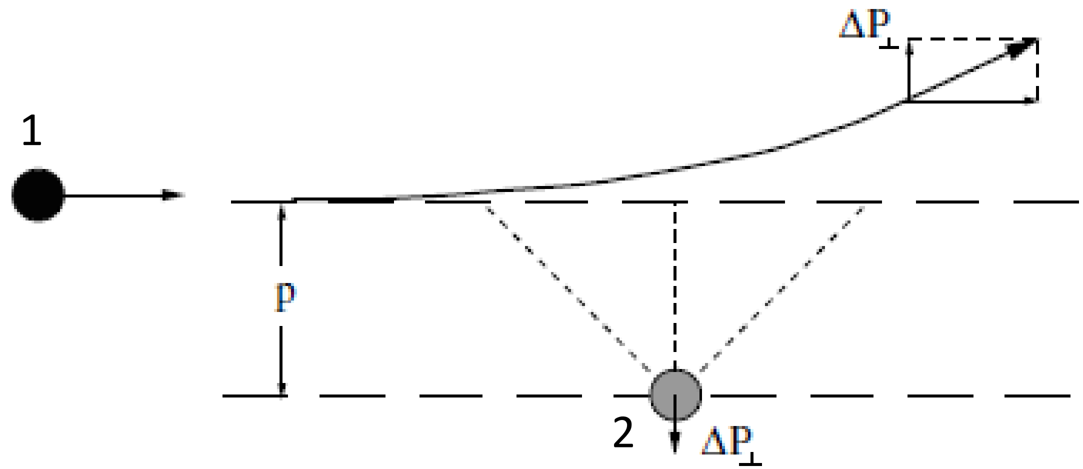
The range or total path length *R* is given for *x* = *l* by setting E(l)=0

$$R_{CSDA} = \int_{0}^{E_0} \frac{dE'}{NS(E')}$$

This estimate of the range, based on CSDA, is valid only in the case of negligible straggling

Classical model of the stopping force

- Model established in 1913 by Niels Bohr → without quantum mechanics and non-relativistic → amazingly correct (foe a particular energy range)
- Let consider a projectile with charge e_1, mass m_1, velocity v (non-relativistic) and a target particle with charge e_2, mass m_2, initially at rest → Coulomb scattering with impact parameter p (distance between the projectile straight-line trajectory and the target particle) supposed to be not too small: « soft » collision



Momentum transfer in the classical model (1)

Assumption: The target particle receives only a small momentum and it can be considered stationary for the duration of the collision.

Momentum transfer* → $$\overrightarrow{\Delta P} = \int_{-\infty}^{+\infty} dt\, \overrightarrow{F}(t)$$

with $$F(t) = \frac{e_1 e_2}{p^2 + (vt)^2}$$

* in Gaussian units

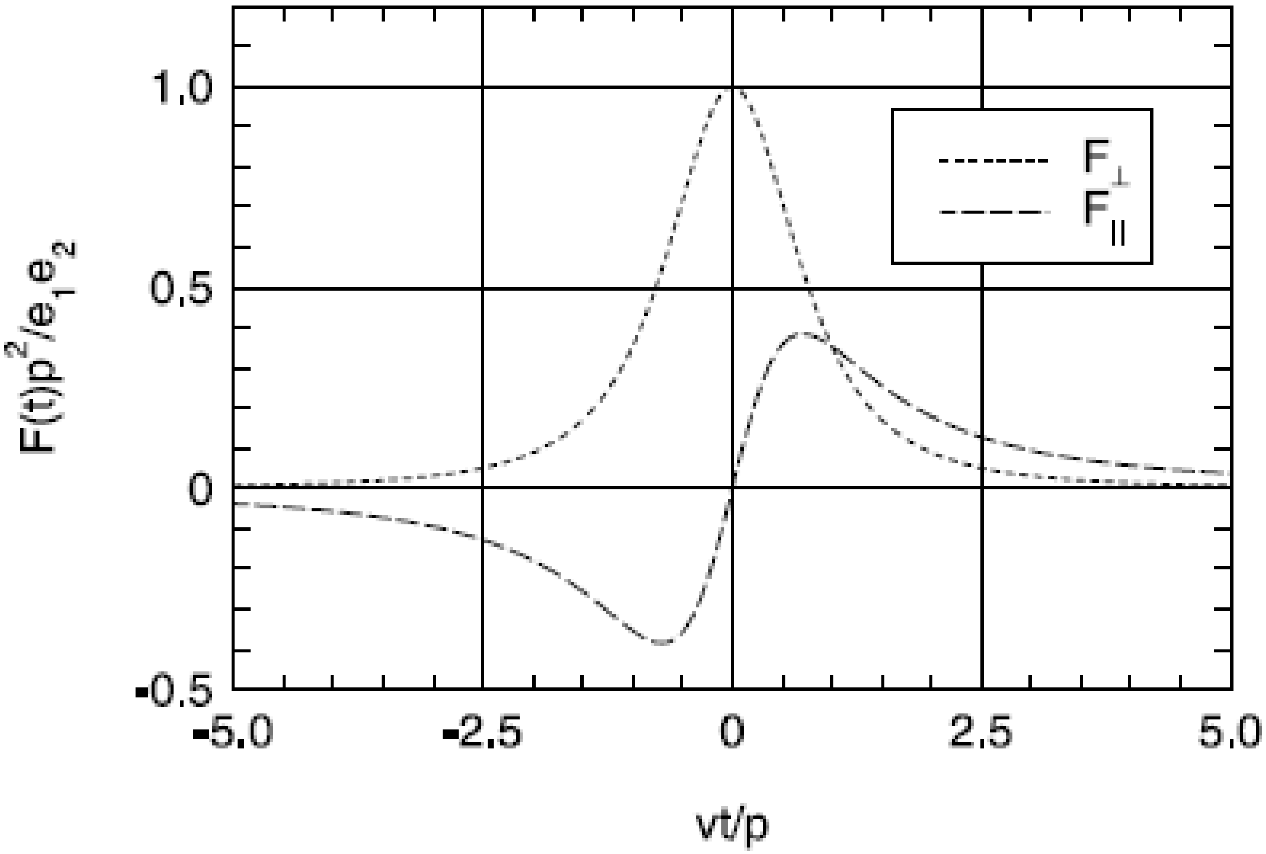
Momentum transfer in the classical model (2)

$$\overrightarrow{F} = F_{\|}\overrightarrow{1_{\|}} + F_{\perp}\overrightarrow{1_{\perp}}$$

$$\Delta P_{\|} = e_1 e_2 \int_{-\infty}^{+\infty} dt \frac{vt}{(p^2 + (vt)^2)^{3/2}} = 0$$

$$\Delta P_{\perp} = e_1 e_2 \int_{-\infty}^{+\infty} dt \frac{p}{(p^2 + (vt)^2)^{3/2}} = \frac{2|e_1 e_2|}{pv}$$

Time dependence of the force



Collision time

The collision time τ can be approximated by

$$\Delta P_{\perp} \simeq F_{max}\tau$$

with $F_{max} = E_1E_2/p^2$, the force at the closest approach (p at $t = 0$)

$$\tau \simeq \frac{2p}{v}$$

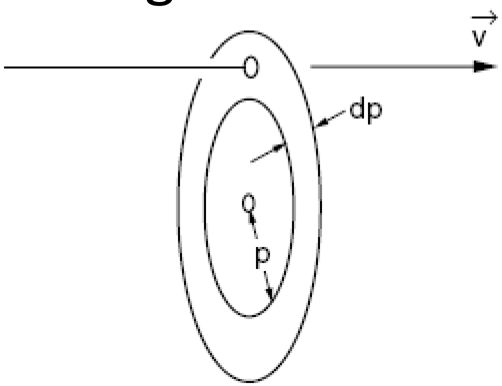
→ the two particles interact effectively over a length $2p$ along the incoming trajectory

Cross section for Coulomb scattering (1)

- The energy transferred from 1 to 2:

$$\Rightarrow T = \frac{\Delta P_\perp^2}{2m_2} \simeq \frac{2e_1^2 e_2^2}{m_2 v^2 p^2}$$

- *T* only depends on the parameter *p* → the number of collisions characterized by a transferred energy between *T* and *T + dT* is equal to the number of collisions characterized by an impact parameter between *p* and *p + dp*
- The cross section of the projectile dσ must be inside the area of the ring:



$$d\sigma = 2\pi p dp = \left| \frac{d(\pi p^2)}{dT} \right| dT$$

Cross section for Coulomb scattering (2)

- We thus find:

$$d\sigma \simeq 2\pi \frac{e_1^2 e_2^2}{m_2 v^2} \frac{dT}{T^2}$$

- It is the correct expression of Rutherford's cross section for Coulomb scattering

Stopping and Straggling: Preliminary Estimates

$$S = \int T \, d\sigma$$

$$W = \int T^2 \, d\sigma$$

$$S \simeq 2\pi \frac{e_1^2 e_2^2}{m_2 v^2} \int_{T_{max}}^{T_{min}} \frac{dT}{T}$$

$$W \simeq 2\pi \frac{e_1^2 e_2^2}{m_2 v^2} \int_{T_{max}}^{T_{min}} dT$$

$$S \simeq 2\pi \frac{e_1^2 e_2^2}{m_2 v^2} \ln\left(\frac{T_{max}}{T_{min}}\right)$$

$$W \simeq 2\pi \frac{e_1^2 e_2^2}{m_2 v^2} (T_{max} - T_{min})$$

$$S \simeq 4\pi \frac{e_1^2 e_2^2}{m_2 v^2} L$$

with the stopping number:

$$L = \frac{1}{2} \ln\left(\frac{T_{max}}{T_{min}}\right)$$

Stopping: Preliminary result

electrons of the target (e): density NZ_2, mass *m* and charge *-e*
nuclei of the target (n): density *N*, masse M_2 and charge Z_2e

$$S_e = \frac{4\pi e_1^2 e^2}{mv^2} L_e \Rightarrow \langle \Delta E \rangle_e \simeq N Z_2 \Delta x \times \frac{4\pi e_1^2 e^2}{mv^2} L_e$$

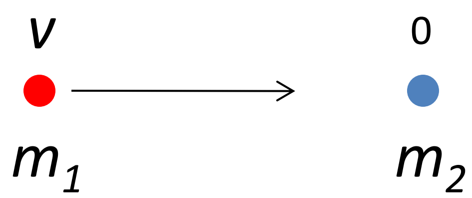
$$S_n = \frac{4\pi e_1^2 Z_2^2 e^2}{M_2 v^2} L_n \Rightarrow \langle \Delta E \rangle_n \simeq N \Delta x \times \frac{4\pi e_1^2 Z_2^2 e^2}{M_2 v^2} L_n$$

$$\Rightarrow \frac{\langle \Delta E \rangle_n}{\langle \Delta E \rangle_e} \simeq \frac{m}{M_2} Z_2 \frac{L_n}{L_e} \quad \text{or} \quad \frac{m Z_2}{M_2} < 10^{-3} \Rightarrow$$

Dominating role of electronic stopping force except for $L_e \simeq 0$ (that happens for small energies)

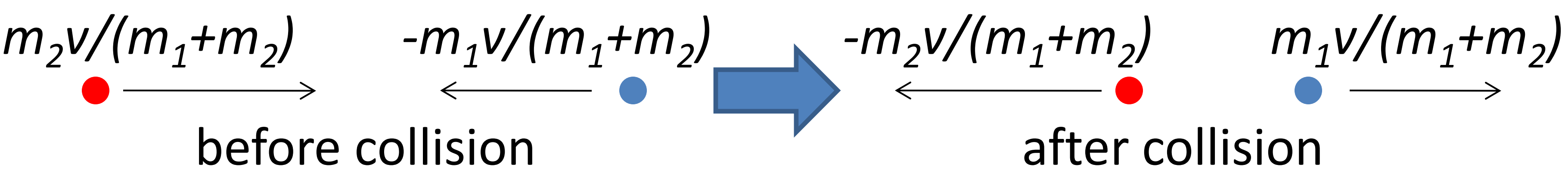
Determination of T_{max} (1)

- We have T_{max} for $p = 0$ → not a soft collision but only a maximum value is searched
- In the laboratory frame →



- In the center of mass (CM) frame (‘) →

$$v_{CM} = \frac{m_1 v}{m_1 + m_2} \qquad v' = v - v_{CM}$$



Elastic collision in CM → modification of the particles direction in CM

Determination of T_{max} (2)

- $v_{2,max}$ for particle 2 (in the laboratory frame) →

$$v_{2,max} = \frac{2m_1 v}{m_1 + m_2}$$

- The maximal transferred energy is →

$$T_{max} = \frac{m_2 v_{2,max}^2}{2} = \gamma E$$

with

$$\gamma = \frac{4m_1 m_2}{(m_1 + m_2)^2} \quad \text{and} \quad E = \frac{m_1 v^2}{2}$$

Consequences of T_{max}

For $m_1 = m_2 \rightarrow \gamma = 1$
For $m_1 \ll$ ou $\gg m_2 \rightarrow \gamma$ small

electron/electron: large energy transfer possible
electron/ion or ion/electron: small energy transfer
ion/ion: large energy transfer possible

Determination of T_{min} (electronic stopping) (1)

- Electronic stopping dominant → determination of T_{min} for a collision with an electron
- For isolated and free electron → $T_{min} = 0$ → logarithmic divergence of the cross section → 2 ways to suppress the divergence →
  - e^- is bound to an atom or molecule in the medium
  - Screening of the Coulomb interaction
- Here → e^- is bound
- Simple model of Thomson → T_{min} is equal to the smallest excitation energy
- Model of Bohr closest to the quantum result

Determination of T_{min} (2)

- Matter is compared to a set of harmonic oscillators → electron = classical harmonic oscillator with pulsation ω_o (period of the oscillator: $2\pi/\omega_o$)
- If slow collision ($\tau \gg 2\pi/\omega_0$) → the harmonic oscillator undergoes an adiabatic modification and the energy transfer is negligible (adiabatic invariance) → the orbit of the electron is only temporarily distorted: initial and final states are equal
- If the interaction happens in a time interval short with respect to the period ($\tau \ll 2\pi/\omega_0$) → the oscillator undergoes an impulse $\sim F \times \tau$

Determination of T_{min} (3)

- With the Bohr condition $\tau \ll 2\pi/\omega_0 \rightarrow$

$$\Rightarrow \frac{2p}{v} \ll \frac{2\pi}{\omega_0} \Rightarrow p_{max} \sim \frac{v}{\omega_0} \Rightarrow T_{min} \sim \frac{2e_1^2 e^2 \omega_0^2}{mv^4}$$

with $$T \simeq \frac{2e_1^2 e_2^2}{mv^2 p^2}$$

- p_{max} is called the adiabatic Bohr radius

Electronic stopping cross section (Bohr model)

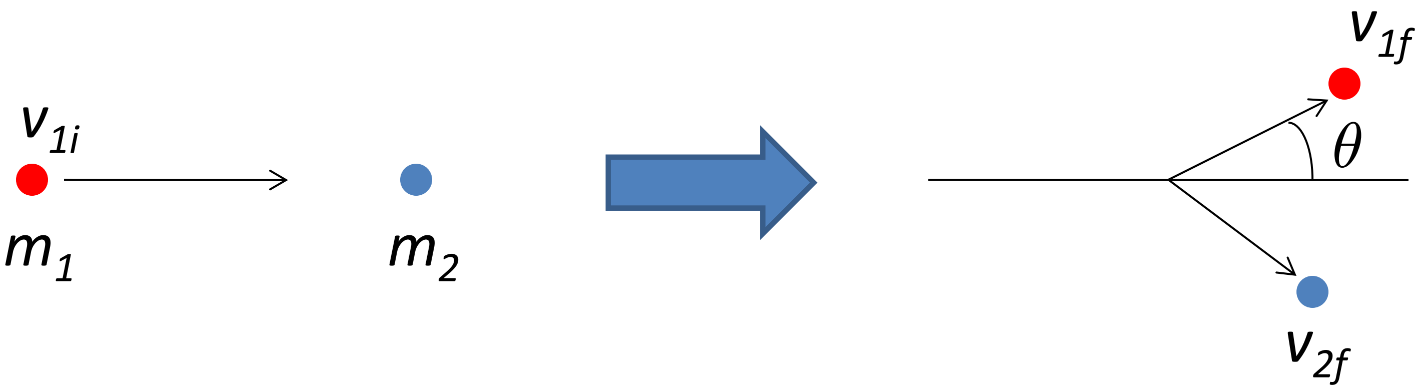
$$S_e = \frac{4\pi Z_2 e_1^2 e^2}{mv^2} L_e$$

with $$L_e = \ln \frac{Cmv^3}{|e_1 e|\omega_0} \quad \text{and} \quad C \simeq 1$$

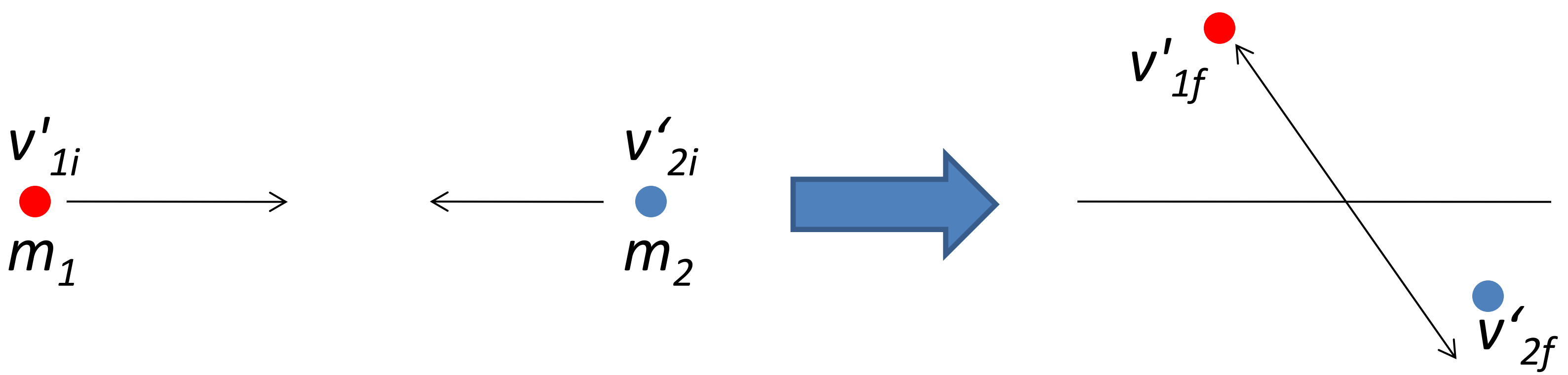
with $L = \frac{1}{2} \ln \left(\frac{T_{max}}{T_{min}} \right)$

Maximum angular deviation (1)

- We consider first $m_2 \leq m_1$
- In the laboratory system →



- In the center of mass (CM) system (') →



Maximum angular deviation (2)

- In CM →

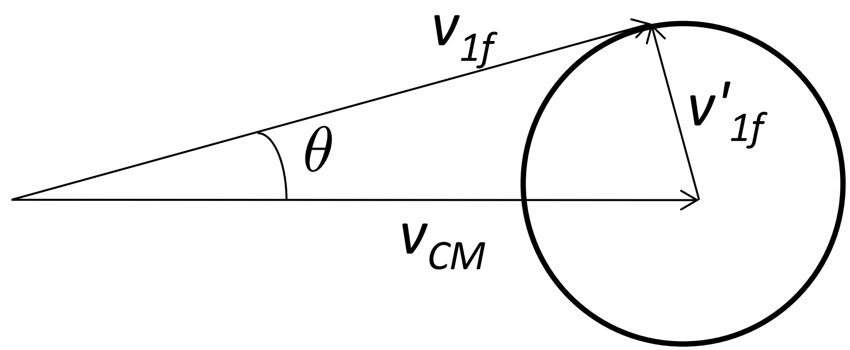
$$v_{CM} = \frac{m_1}{m_1 + m_2} v_{1i} \quad \text{and} \quad v' = v - v_{CM}$$

$$\Rightarrow \quad v'_{1i} = v_{1i} - v_{CM} = \frac{m_2}{m_1 + m_2} v_{1i}$$

- Elastic collision in CM → modification of the particles directions → $v'_{1f} = v'_{1i}$ →

$$v'_{1f} = \frac{m_2}{m_1 + m_2} v_{1i}$$

Maximum angular deviation (3)



- As v_{CM} is fixed and v'_{1f} can take any orientation → we need to find the orientation of v'_{1f} such as θ is maximum →

$$\sin\theta_{max} = \frac{v'_{1f}}{v_{CM}} = \frac{m_2}{m_1}$$

- If $m_2 \geq m_1$ → $\theta_{max} = \pi$

Maximum angular deviation (4)

electron/electron: large deviations possible $\theta_{max} = \pi/2$
electron/ion: very large deviations possible $\theta_{max} = \pi$
ion/electron: small deviations
ion/ion: large deviations possible (depending on m_1 and m_2)

First conclusions about basic considerations

- For incident ions:
  - electronic losses dominating
    - → small energy transfers
    - → small angular deviations
  - nuclear losses (collisions with nuclei) rare
    - → only occur for a few projectiles (and for small energies)
    - → large energy transfers possible
    - → large angular deviations

⇒ **Rectilinear trajectory with continuous energy losses**

First conclusions about basic considerations (2)

- For incident electrons:
  - electronic losses dominating
    → large energy transfer possible
    → large angular deviations possible
  - nuclear losses:
    → very small energy transfer
    → very large angular deviations possible (backscattering)

⇒ Curled trajectory with large energy losses possible